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Modeling Inter-Country Spatial Financial Interactions with Graphical Lasso: an Application to Sovereign Co-Risk Evaluation

GIUSEPPE ARBIA, RICCARDO BRAMANTE, SILVIA FACCHINETTI & DIEGO ZAPPA

ABSTRACT:
We propose a model to extract significant risk spatial interactions between countries adopting the Graphical Lasso algorithm, used in graph theory to sort out spurious conditional correlations. In this context, the major issue is the definition of the penalization parameter. We propose a search algorithm aimed at the best separation of the variables (expressed in terms of conditional dependence) given an a priori desired partition. The case study focuses on Credit Default Swap (CDS) returns over the period 2009–2017. The proposed algorithm is used to estimate the spatial systemic risk relationship between Peripheral and Core Countries in the Euro Area.

KEYWORDS: *Regional financial contagion, Spatial conditional dependence; Systemic risk; Network dependence*

JEL CLASSIFICATION: C13; C51; C61; G01

1. Introduction

Spatial interaction models have a long tradition in the regional economic literature. Starting from the publication of the first prototype of a gravity-type model (Isard, 1960; Wilson, 1970), the literature has evolved dramatically in recent decades in various directions. It now includes more realistic and complex models to answer the current and future demand coming from a wide range of applied fields that goes beyond the traditional fields of regional studies: for example, finance and social network analysis to name but one. One challenging issue in this area is represented by the measurement and modeling of spatial, network dependence and of network autocorrelation (Griffith, 2007; LeSage & Pace, 2008; Bavaud, 2016; Patuelli & Arbia, 2016).

In this paper, we propose an original approach based on Graphical Lasso (GLasso; see Friedman et al. 2008) to investigate the spatial financial interactions between countries from a systemic risk perspective.

The recent debt crisis in the Euro Area has turned researchers' attention to the measurement of sovereign default risk and the spatial propagation of systemic risk. As the sovereign debt crisis has evolved and bond yields have increased, the interest in credit risk protection for Euro sovereign borrowers via Credit Default Swaps (CDS) has undoubtedly grown. CDS co–movements among countries can help us to understand how correlations of default probabilities evolve over time and are diffused in the geographical space so as to provide considerable insights on the possible direction of future defaults. However, while CDS quickly captures the market information risk content (Damodaran, 2003) and the corresponding returns can then provide a more timely early warning to detect updated changes in country–specific risk co–movements (e.g. in the Eurozone Periphery; see Buchholz & Tonzer, 2013; Elkhaldi *et al.*, 2014), on the other hand, it is well documented that CDS market movements can be vulnerable to market information or financial speculation and thus might be a misleading proxy of country risk dependence (Revoltella *et al.*, 2010). For example, the main assumption, common to many studies in this field, is the conditional independence between Core and Peripheral Countries (ECB, 2016), an hypothesis that is difficult to be sustained in most regional economic applications.

To overcome this criticism, in this paper we propose the use of GLasso to focus the study only on the relevant sovereign risk co–movements within the Euro Area, analyzing the CDS returns of 17 European countries over the period 2009–2017, a period that includes the European sovereign debt crisis as well. For an application of GLasso in the financial field see, for example, ECB, 2013; Goto & Xu, 2015; Huang & Shi, 2011.

More specifically, the aim of this paper is twofold. First of all, our contribution focuses on the choice of the penalty parameter within a GLasso framework. As it is known, the penalization parameter is typically used to mitigate the effect of spurious cross–sectional/longitudinal dependence among the variables involved in order to shed light on the expected or hidden true spatial linkages. This is a relevant point in the estimation process of the network used to represent the interrelationships among European countries.

The choice of this crucial parameter, however, is in most cases subjective and expertise-driven. To avoid this subjectivity, we propose modifying the calibration search algorithm in Friedman et. al (2008) by searching for the minimum of the absolute difference between the CDS returns expected precision matrix and its estimate after penalization. This method allows us to keep at the minimum level the sparsity of the precision matrix after penalization. Secondly, we analyze cross-country contagion effects by investigating – in a rolling framework – the characteristics of the degree of connectivity of the examined countries, in the graph obtained after penalization.

The rest of the paper is organized as follows. Section 2 reports some details about the GLasso algorithm and the procedure developed to calibrate the penalization parameter. In light of the current sovereign debt crisis, the application proposed in Section 3 refers to Euro Zone systemic risk analysis. Section 4 reports conclusions.

2. Methodology

2.1 Measuring system relevance in a spatial network

Let $\boldsymbol{X}=\{X_1,X_2,...,X_p\}$ be a p–multivariate random variable. Let Σ and $\Theta=\Sigma^{-1}$ be its covariance and precision matrix, respectively. Using algebra, it may be shown that Θ is proportional to the partial correlation matrix with (i, j) element

$$\rho_{(i,j)|\{X\backslash(X_i,X_j)\}} = \frac{-\theta_{ij}}{\sqrt{\theta_{ii}\theta_{jj}}} \tag{1}$$

where $\theta_{ij} \in \Theta$. Equation (1) represents the correlation coefficient between X_i and X_j conditional on the remaining variables. The matrix Θ can be effectively used to characterize the spatial dependence of the variable of interest through the associated graph (Edwards, 2000).

In brief, a simple undirected graph, G = (V,E), is a mathematical structure consisting of a finite set, V, of vertices (or nodes) – corresponding to the variables in the model – and a finite set E of undirected edges (or arcs) between the vertices. The non–zero correlation between X_i and X_j, conditional on the remaining variables, is equivalent to the presence of an edge between the two nodes (Whittaker, 1990). By assuming the p variables to be jointly Gaussian, if $\theta_{ij} = 0$ then the corresponding variables are conditionally independent of the others, and no edge between vertices i and j is drawn (Banerjee *et al.*, 2008; Mazumder & Hastie, 2012a).

The key role played by the matrix Θ is obvious, which justifies the many approaches proposed to estimate it efficiently and robustly with respect to abnormal deviations (Bickel & Levina, 2008; Ledoit & Wolf, 2004). In particular, most of the contributions are related to the efficient estimation of the inverse covariance matrix even in the presence of quasi–collinearity, while other solutions directly focus on the estimation of each element of the precision matrix (see, for example, Dempster, 1972; Ledoit & Wolf, 2012; Pourahmadi, 2011; Wong, 2003).

Aiming at the selection of the best graph, standard inference is often used, based either on likelihood or on penalized model selection criteria. In graphical Gaussian models, the most widely used approach is the stepwise forward–selection, which starts from some initial model and then progressively adds or removes edges until some optimality criterion is fulfilled. In each step, the edge selection is typically performed using some test of significance. The problem with such an approach is that the stepwise procedure is often computationally complex and does not correctly account for the multiple comparisons which are involved (Edwards, 2000). An alternative approach was presented by Meinshausen & Buhlman (2006), who proposed a neighborhood selection procedure which estimates the conditional independence restrictions separately for each node in the graph.

A more recent approach, which combines model selection with parameter estimation, is represented by the Graphical Lasso algorithm, which uses a regularization framework to estimate the covariance matrix under the assumption that its inverse is sparse (Friedman et al., 2008)[1].

2.2 The GLasso

The methodological root of the GLasso is the maximum likelihood principle. The aim of such a procedure is to improve the interpretability of results by adding a penalization while running the estimation routine. In doing so, we undermine the MLE logic, willing to pay a price in terms of methodological "pureness" in exchange for more regular, stable outputs.

In detail, the problem is to estimate a covariance matrix, or almost equivalently the related precision matrix, by removing those elements that likely represent spurious correlations. The way to achieve this is by introducing a penalization into the maximum likelihood estimation of the precision matrix using an L_1 penalty function over nonnegative definite matrices Θ:

$$arg\max_{\Theta>0}\{\log\det\Theta - \mathrm{tr}(S\Theta) - \lambda||\Theta||_1\} \tag{2}$$

where $\|\Theta\|_1$ is the L_1 norm of Θ, S is the empirical covariance matrix and λ a scalar parameter that controls the size of the penalty[2]. The smaller the value of λ is, the higher will be the degree of spatial dependence and the density of the graph.

Friedman et al. (2008) developed a fast algorithm to solve the optimization problem[3] . This is a block-coordinate descent-type algorithm based on the general idea of the Lasso algorithm (Tibshirani, 1996) to estimate recursively a single row and column of Θ in each iteration. Banerjee *et al.* (2008) show that the optimization problem is convex, considering the estimation of Σ rather than Σ^{-1}. In addition, Witten *et al.* (2011) have discussed some of the related computational issues.

The penalized maximum likelihood estimation for Σ can be computed $\forall\, \lambda \geq 0$. In particular, if l_{λ_0} and l_{λ_1} represent the log-likelihood for $\lambda_0 < \lambda_1$,

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1 In the paper by Mazumder *et al*. (2012b), two alternatives to the Graphical Lasso algorithm – the p-GLasso and the dp-GLasso algorithms – are proposed.

2 It has been shown that the L_1 penalty function removes insignificant variables by forcing Θ to be as sparse as possible (Tibshirani, 1996).

3 This procedure is implemented in the `glasso` *R* package (see Friedman *et al.*, 2010, for details). For high dimensional undirected graph estimation, the `Huge` *R* package can be used as a more computationally efficient alternative (Zhao *et al.*, 2012).

it can be shown that $l_{\lambda_0} \geq l_{\lambda_1}$, thereby allowing the likelihood ratio test statistic

$$lrt = -2\,(l_{\lambda_1} - l_{\lambda_0}) \qquad (3)$$

to be used. Having fixed λ_0, e.g. $\lambda_0 = 0$, the best choice for λ could be the largest λ_1 that makes *lrt* not significant. This approach has been shown to induce a strong sparsity into Σ and, since *lrt* is only asymptotically χ^2 distributed, works only when the dimension of the multivariate problem is very large.

In summary, the main features of the GLasso are:

- the variances are not affected by the application of this procedure; only covariances are reduced;
- the parameter used to calibrate the strength of the filtering action of λ is arbitrary and it is not possible to establish a "best practice" in an absolute sense. To achieve sparsity, the greater is λ, the stronger will be the structural and unavoidable dependence among variables;
- because of penalization, the MLE estimate properties are not always granted.

2.3 Calibration of the penalization parameter

According to Equation (3), in principle we should expect a starting configuration of the precision matrix characterized by exact zero constraints. However, such a strict structural dependence configuration (from here on represented by Θ_H) could be too restrictive from an empirical point of view. In fact, because of randomness, even if exact conditional independence is expected, a weak dependence might be typically observed. Therefore, to allow for sufficient parametric flexibility a configuration with constraints not exactly equal to zero is more desirable. For these reasons, we propose changing the strategy in Equation (3) by defining a search algorithm to solve:

$$\min_\lambda ||\Theta_H - \widehat{\Theta}_\lambda||_1 \qquad (4)$$

where Θ_H and $\widehat{\Theta}_\lambda$ represent the expected/desired precision matrix and the estimate of Θ after penalization, respectively. The choice of the L_1 norm is in accordance with the Lasso principle in (2). As mentioned above, the main advantage of Equation (4) is its flexibility: it allows us to define a conditional dependence structure without the constraint that the solution must exactly match that structure. Starting from a covariance matrix, we may even partition the multivariate phenomenon by setting only some of the entries of the precision matrix to zero. The solution to Equation (4) may allow some elements of Θ to be different from zero. This appears useful, for example, in the empirical analysis that we will discuss in Section 3. In this case, in fact, due to the financial characteristics of the data, we expect a sort of structural conditional independence between Peripheral and Core Countries, even if a small dependence due to common market factors may exist.

3. Empirical analysis

This section presents an implementation of Equation (4) as well as a case–specific strategy for the computational issues mentioned in Section 2. Such a solution allows us to highlight relevant connections among the variables and to display their dynamics over the historical period in question.

3.1 The CDS market and systemic risk transmission mechanism in the European Union

Two of the major contributions to the study of Sovereign CDS[4] markets are by Longstaff (2010) and Pan & Singleton (2008). Among the fundamental factors in these studies, growth prospects and forward-looking fiscal indicators appear to be particularly relevant. The role of fundamentals is particularly strong for high–debt and low–growth countries (e.g., Portugal, Italy, Ireland, Greece, Spain, also called PIIGS Countries). For both descriptive purposes and quality picture representation, Figure 1 depicts the corresponding CDS trajectories from 2009 to October 2017, focusing only on the 4 major developed countries in the EU (Italy, Spain, Germany and

[4] Thanks to the referees' comments, as a benchmark, we have applied the proposed methodology to sovereign bond returns, known to be a close proxy of the CDS returns. The results are very similar. For this reason, we report comments only regarding CDS.

France).

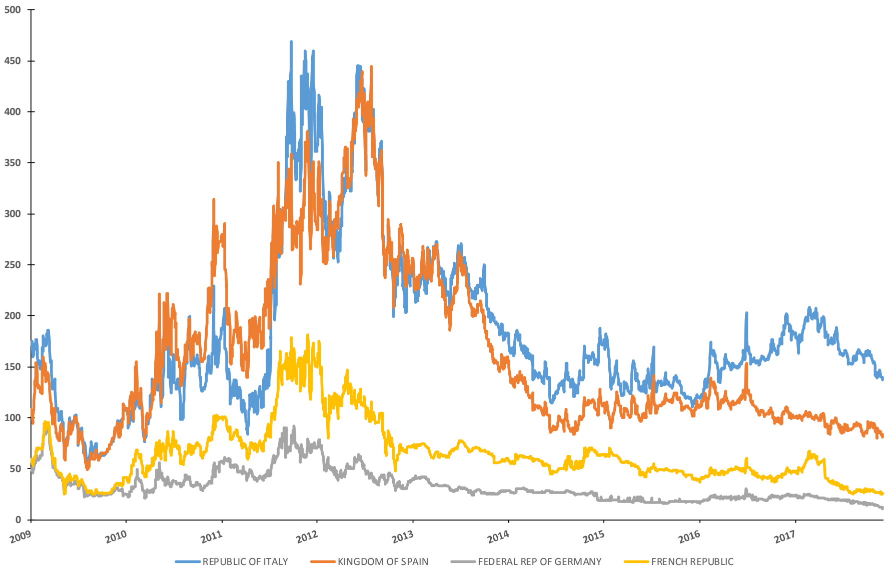

Fig. 1. Ten–year Sovereign CDS spreads

The pattern of the series in the graph unambiguously shows the effect of the crisis on Italy and Spain, and a much more moderate effect on France and Germany. CDS spreads have dramatically risen from 2011 to 2013, a period affected by wide changes in global risk aversion. Firstly, CDS spreads started moving up very sharply in 2008 and 2009 during the global financial and economic crisis. From the first half of 2010 on, around the early stage of the Euro Area crisis, spreads began moving upwards, continuing to widen sharply in 2011. The sharp decline started in the second half of 2012 due to the policies of the European Central Bank, with a subsequent stabilization around a roughly flat trend.

The sovereign debt crisis in the Eurozone has increased the attention to country risk and generated a large and rich literature (e.g., Dieckmann & Plank, 2012; Fontana & Scheicher, 2010; Lucas *et al.*, 2014; Muratori, 2015). The main issue is that, quite often, the statistics used to evaluate risk–dependence among countries are based on covariance or correlation measures (ECB, 2016). However, it is well known that these statistics measure the dependence between two variables and, if computed in the presence of other variables (covariates), appropriate corrections should be applied to highlight the real dependence between each pair of countries by filtering the dependence on latent common variables. This step is very tricky since it may significantly affect the sign and the intensity of the spatial financial interaction between countries. This effect may be further inflated in all those procedures where the inverse of the covariance matrix is used (Kourtis, 2012). In fact, examples of this are the studies on CDS returns when the dependence among the default risks of a panel of countries is evaluated: cross–sectionally, CDS are naturally correlated, but dependence is due both to country specific variables (as EU Countries must independently control local sovereign debt) and to common fiscal and monetary rules. This inflates the dependence and produces in some cases even significant, although spurious, partial correlations.

To appreciate this effect, covering 17 countries in the Eurozone, we have gathered CDS daily data between 2009 and October 2017 related to EUR denominated "contracts" with a ten-year maturity[5]. This includes five Peripheral Countries (Greece, Italy, Ireland, Portugal and Spain), five Core Countries (Austria, Belgium, France, Germany, and Netherlands), and the seven "Other" remaining countries (Cyprus, the Czech Republic, Finland, Latvia, Lithuania, Slovenia, and the Slovak Republic), which subsequently (except for Finland) adopted the Euro currency[6].

By using CDS returns, Figure (2) shows: 1) in the diagonal the series of CDS returns[7]; 2) in the upper triangular matrix the correlation coefficient for each cross of row and column; 3) in the lower triangular matrix the related partial correlation coefficient, according to (1). To avoid unnecessary complexity, the matrix reports results for just a selection of countries (GR=Greece, IT=Italy, IE=Ireland, PT=Portugal, ES=Spain, FR=France, DE=Germany, CY=Cyprus).

For example, the unconditional correlation between DE and IT is 0.39, while the conditional estimate drops to 0.03, showing that the net country risk specific relationship is significantly lower. On the other hand, the unconditional correlation between IT and ES is 0.8, while the conditional estimate remains quite relevant at 0.56. In general, moving from the conditional to the unconditional correlations within the matrix block of PIIGS (Peripheral) countries or the Core countries, positive and significant correlations may be found in many pairs as evidence of the intra-dependence of

5 Source: Bloomberg.
6 Luxembourg is excluded from the analysis since no data are available.
7 For GR we do not have data from March 2012 to December 2013.

the CDS returns. The only exception is the conditional correlation between GR and the other Peripheral countries, which is quite close to zero. This may represent empirical evidence that contagion is largely related to purely speculative attacks and not to risk co-movements, e.g., common economic or financial strategies. Analogously, the conditional correlation between the Peripheral-Core block reports values which in some cases are quite close to zero.

The correlations with CY, another country that experienced a sovereign debt crisis, remains substantially unchanged when we move from the unconditional to the conditional vector. The same patterns can be found when we consider the countries excluded from Figure 2.

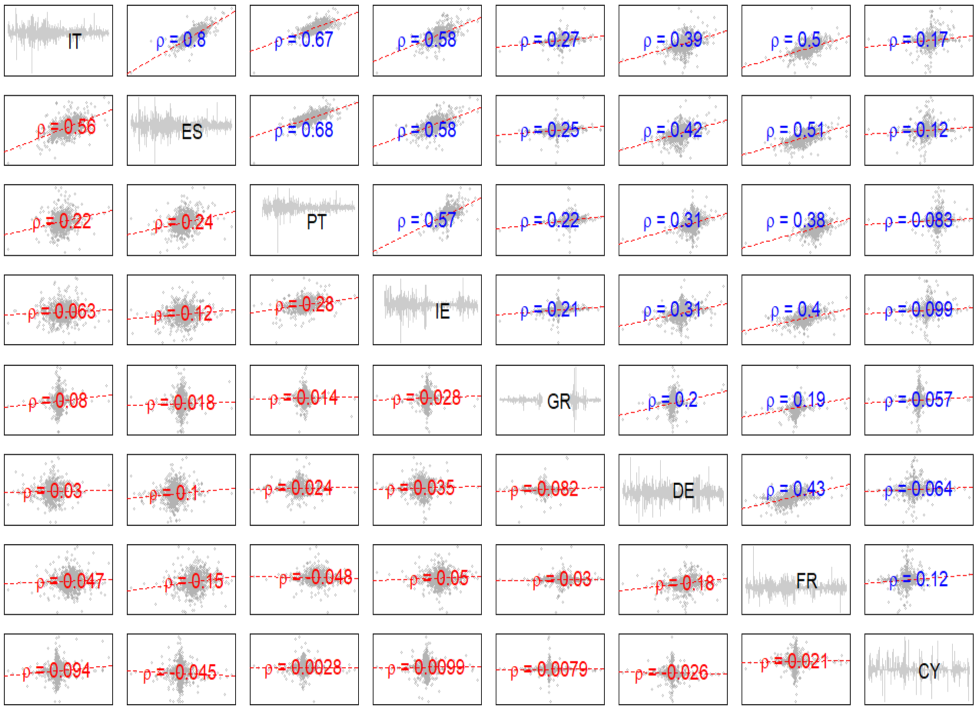

Fig. 2. Ten–year Sovereign CDS returns; time series (diagonal), correlations (upper triangular matrix) and partial correlation (lower triangular matrix)

Two open issues must be considered. First, the conditional correlation still suffers from the marginal effects of spurious correlations; moreover, what we have described concerns the relationship between pairs of countries leaving out the effect of the others: what happens when we wish to study jointly the multivariate phenomenon? To eliminate this problem, an appropriate solution is to add a penalty into the covariance estimation step according to the GLasso principle by allowing the estimates of the corresponding inverse matrix to be as sparse as possible and by reinforcing the conditional uncorrelation, if necessary. Second, in this section we have considered a very long and heterogeneous time series window: how do we consider the effect of heteroskedasticity and time varying correlations? To filter this effect, we will study the application of GLasso using a rolling window-size schema.

3.2 An application of the Graphical Lasso to the systemic risk spatial propagation in the Euro Area

This section studies the application of GLasso over the rolling windows of size 150, 200 and 300 days[8]. For simplicity's sake, we report results only for the 150 window size.

Careful consideration to missing data should be given when interpreting the results since two of the countries (Greece and Finland) included in the analysis suffer from clusters of missing observations. In addition to the time instances where information is strictly not available, the time periods with persistent subsequent daily CDS returns equal to zero should also be considered missing. The latter are evidence that either the system was unable to record for some reason the value of CDS or there were no trading data. Many ad–hoc approaches to these problems are available in the literature (Puliga *et al.*, 2014). Due to the type and the significant rate of missing information in the CDS time series, for each time window we have filtered from the analysis the countries with both/either a lack of data equal to at least 90% of the rolling period and/or with zero consecutive daily CDS returns for at least 90% of each rolling period.

The Core–Periphery dualism in the Eurozone is well known, highlighted by the recent sovereign debt crises. To reflect the evolution of systemic risk, a way to dynamically assess the degree of connection among the Core, Peripheral and "Other" countries is needed, which hinges on the dynamic application of the GLasso algorithm to estimate at time t the preci-

8 In general, the choice of the window length involves balancing two opposing factors: on the one hand, a larger window could involve changing the data generating processes, whereas a shorter period implies a smaller data set available for estimation. For the purposes of the empirical investigation proposed in this paper, the choice of the three proposed windows seems to be a good compromise between the two factors.

sion block matrix Θ_t as follows:

$$\Theta_t = \begin{bmatrix} {}_{PP}\Theta_t & {}_{PC}\Theta_t & {}_{PO}\Theta_t \\ {}_{CP}\Theta_t & {}_{CC}\Theta_t & {}_{CO}\Theta_t \\ {}_{OP}\Theta_t & {}_{OC}\Theta_t & {}_{OO}\Theta_t \end{bmatrix} \tag{5}$$

where t is the time index and subscripts P, C and O indicate Peripheral, Core and remaining (Other) countries, respectively.

To apply Equation (4) we need to define Θ_H as:

$$\Theta_H = {}_{h,k}\Theta_{\lambda_{0,t}} - \mathrm{diag}({}_{h,k}\Theta_{\lambda_{0,t}})$$

where $\lambda_{0,t}$ is the tuning parameter at the initial step, which is applied only to the off-diagonal elements of the original precision matrix[9] (i.e., it may be any non-negative or fixed value, e.g., $\lambda_{0,t}$=0 when we assume as the starting point the configuration of the graph without penalization). ${}_{h,k}\Theta$, for h,k=1, 2, 3, represents the h,k matrix block of Θ_t. The term ${}_{h,k}\Theta$ may even refer to more than one block; e.g., h=1, k=1,2 represents the two leftmost blocks of the first row.

According to financial assumptions, a prior constraint could be ${}_{PC}\Theta=\mathbf{0}$. We have tried in our application to limit as much as possible any subjective choice to assess ex–post what financial theory suggests. To achieve this aim, we have changed Equation (4) by searching for a value of λ_i such that for any rolling window:

$$0.5\left|\left\|\Theta_H - \left({}_{h,k}\widehat{\Theta}_{\lambda_{i,t}} - \mathrm{diag}\left({}_{h,k}\widehat{\Theta}_{\lambda_{i,t}}\right)\right)\right\|_1 - \left\|\Theta_H - \left({}_{h,k}\widehat{\Theta}_{\lambda_{i-1,t}} - \mathrm{diag}\left({}_{h,k}\widehat{\Theta}_{\lambda_{i-1,t}}\right)\right)\right\|_1\right| \le \varepsilon \tag{6}$$

where $\lambda_0 < \lambda_{i-1} < \lambda_i$ and ε is the algorithm tolerance level chosen subjectively by the user to guarantee convergence[10]. The rationale in Equation (6) is to avoid the selection of large λ, which may artificially inflate sparsity in the precision matrix. The larger the value of λ, the stronger will be the shrinkage effect and the greater the L_1 distance between Θ_H and its estimate. This enables us to extract graphs with a good balance of sparsity and density. Convergence is assured at least for $\lambda\to\infty$. In that case, GLasso will force the precision matrix to be diagonal, in which case the difference with

[9] The diagonal elements of each block are left unchanged to preserve specific risk related to each country.

[10] To ensure fast convergence of the algorithm, ε is set, by default, to 10^{-4}.

Θ_H will not depend on the restricted conditional correlations. In our case study, optimization has been applied by considering three different contexts:

1) h=1 and k=1,2; i.e., only the two blocks $[{}_{PP}\Theta_t \;\; {}_{PC}\Theta_t]$, related to Peripheral Countries and their connections with the Core ones;
2) h=1,2 and k=1,2; i.e., the 2x2 block $\begin{bmatrix} {}_{PP}\Theta_t & {}_{PC}\Theta_t \\ {}_{CP}\Theta_t & {}_{CC}\Theta_t \end{bmatrix}$, related to Peripheral Countries, their connections with the Core block and the Core block;
3) $\forall h, k$; i.e. the whole matrix.

Since values in the penalized precision matrix are proportional to the precision matrix with λ set to zero (i.e., the minimum in (4) would be exactly zero and the search algorithm could not start), the initial value of the penalty parameter λ_0 is empirically set at 10^{-6}; in other words, arbitrarily small but not exactly zero.

As reported in Section 2, we aim at studying when the *PC* block in Θ_t is zero (e.g., the Peripheral and Core Countries are conditionally independent and no co–risk between the two groups is considered). The trajectory of λ_t that tunes this block to zero[11], along with the corresponding yearly moving average, is reported in Figure 3 on the basis of month end data[12]. The evolution of λ_t and, remarkably, the sharp drop during the mounting sovereign debt crisis, when the debt of some Eurozone economies looked unsustainable, suggests that the penalization parameter dimension can be used as an indicator of the intensity of the crisis. Moreover, while 2014 seems to represent a lower boundary of the crisis, the aftermath of a slowly but steadily increasing pattern indicates that the sovereign debt crisis seems far from a final resolution.

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[11] Note that in the algorithm we have set the constraint to leave active at least one connection within the *PP* and the *CC* block.

[12] This type of data frequency – instead of the daily one – is used throughout this section, as it seems the most appropriate for the data under study to smooth the "erraticness" displayed when daily data are used.

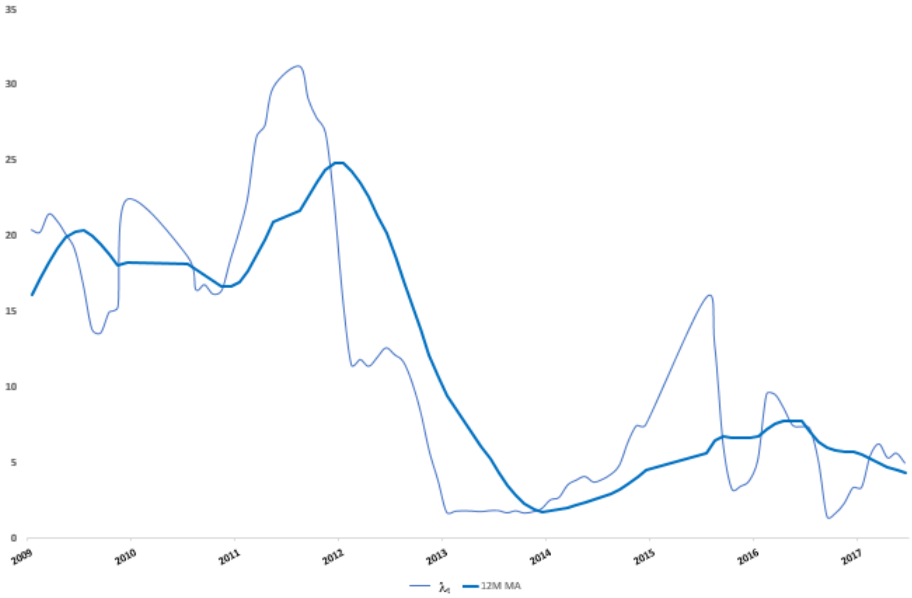

Fig. 3. Minimum λ_t values for obtaining a sparse ${}_{2,1}\Theta_t$ precision matrix

A similar pattern was observed when examining the dynamics of λ_t according to Equation (6). Figure 4 displays the results by comparing the MLE and L_1 estimates.[13]

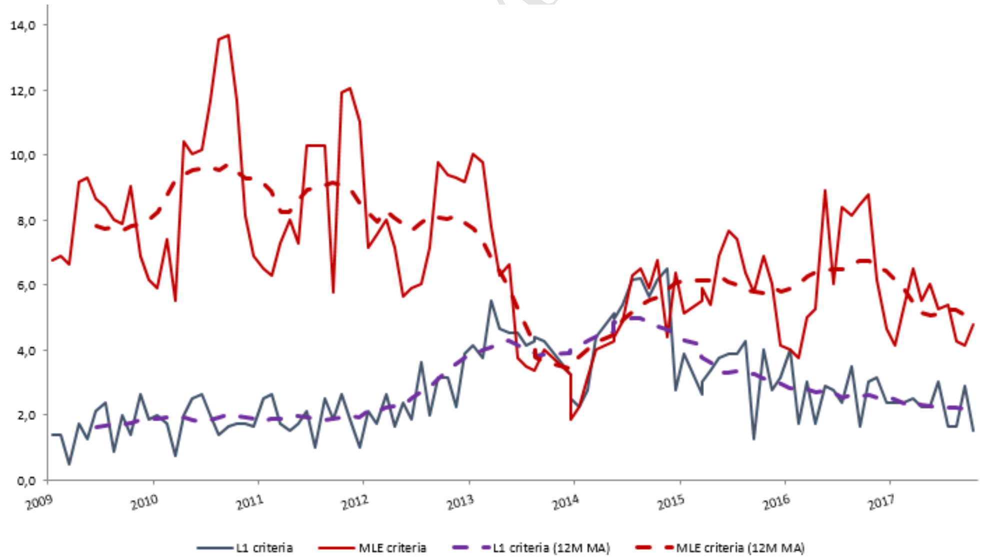

Fig. 4. λ_t comparison: *MLE* versus L_1 criterion

In contrast, the steadily decreasing λ_t pattern in the aftermath of 2014, when the L_1 criterion is considered, provides evidence that the ECB succeeded in reducing market tensions, which resulted in a significant ongoing reduction of systemic risk.

Moreover, the L_1 criterion seems to highlight better the dynamics of spatial

[13] The dashed lines are the corresponding yearly moving averages.

interactions among countries. As an example, Figure 5 shows the graphs obtained by using the best solution to Equations (3) and (4), averaged by year: that is, the application of L_1 criterion (Figure 5A) and MLE (Figure 5B), referring to the years from 2010 to 2017.

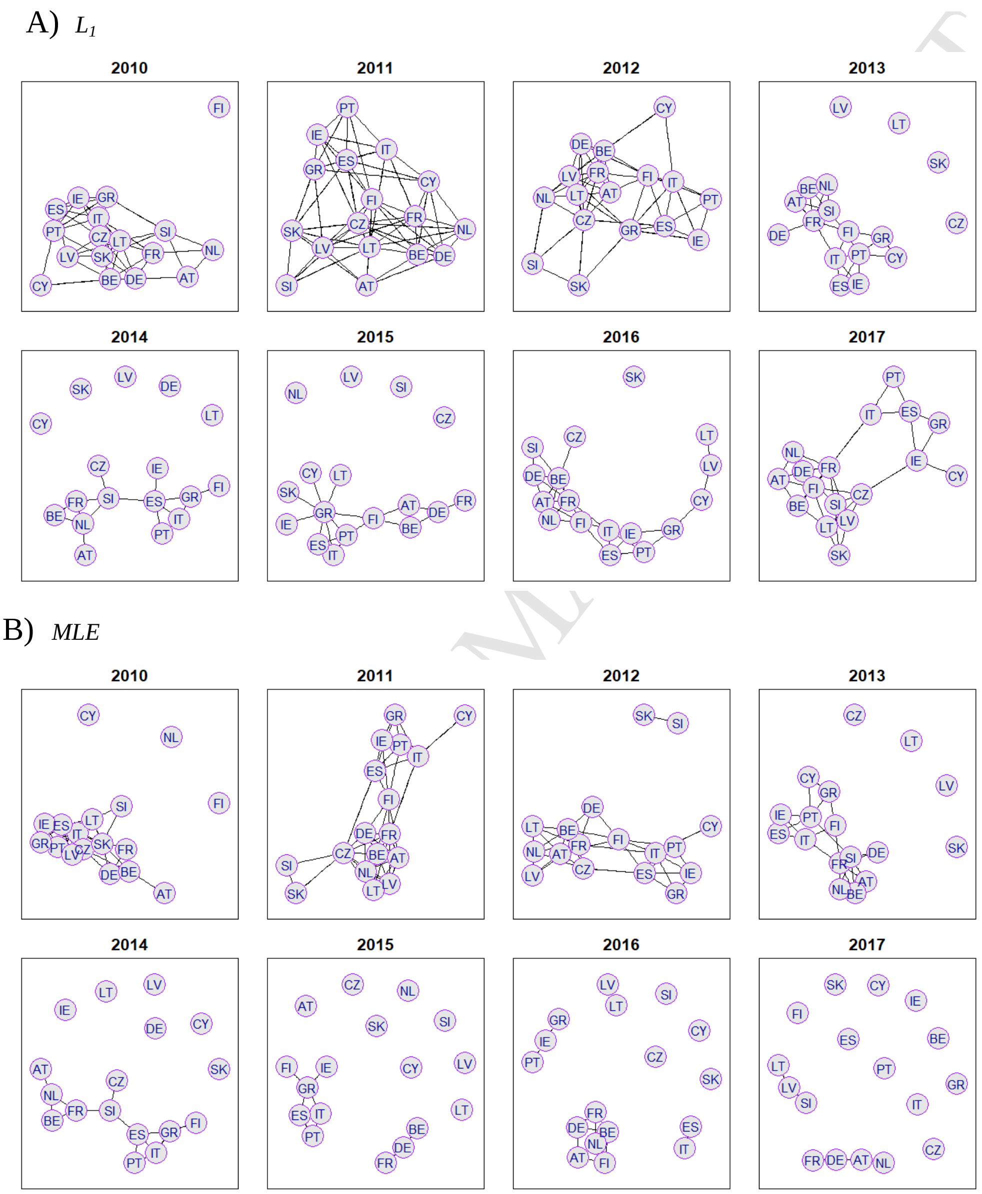

Fig.5. Network per year with the application of the average best penalization solution by using L_1 (A) and MLE (B), referring to the years from 2010 to 2017

Figure 5 shows how the country connections have changed year by year. It seems that *MLE* tends to separate the countries stronger than L_1 does. The tendency to create a cluster within the Peripheral and Core countries is well explained by most of the graphs. Looking at the solution obtained with L_1, this evidence is much clearer. Specifically, from 2010 to 2012, connections increased exactly during the peak of the crisis and with the sudden increase in spreads (see Figure 1). Subsequently, a moderate conditional independence is observed until 2015, when some countries needed to restructure their debts or use additional fiscal constraints to reduce its rate of increase. Note that (see Figure 4) during these years the MLE and L_1 solutions are roughly the same. Beginning in 2016, a much more markedly renewed country dependence can be observed even with moderate CDS risk levels, showing the increasing tendency of many countries to share common economic and fiscal strategies.

4. Conclusions

This paper exploited the Graphical Lasso algorithm to extract significant spatial interactions among geographical regions by showing, in particular, its ability to describe the Euro Countries regional dynamics of CDS returns from a systemic risk perspective. A calibration criterion to identify the best regularization parameter along time and cross–sectionally has been proposed.

The application to CDS returns in the Euro area shows that the proposed method allows us to extract the relevant systemic risk contributions and identify the most relevant spatial interactions, thus lowering the network dimension and isolating spurious relationships. We also show that the penalization parameter can be used as an indicator of the intensity of the crisis, which helps to identify country–specific early warnings and the geographical path of its contagion.

The suggested procedure is very general and it may be used in all regional contexts where a natural segmentation of a multivariate phenomenon is present. Furthermore, the ability of the method to identify significant spatial interactions may prove particularly useful in reducing the dimensionality and the computational effort employed in the analysis of very large datasets, an issue that has been identified as one of the most challenging current research problems in spatial interaction modeling (Arbia & Patuelli, 2016).

The empirical analysis highlights that the most interconnected nodes (countries) change over time as the geographical contagion takes place. We also show that the proposed optimization algorithm describes the dynamics of the relationships between Peripheral and Core Countries better than does the standard MLE techniques through the analysis of the Θ_t matrix structure.

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Highlights

- Study significant risk spatial interactions between countries
- the case study explores the spatial systemic risk relationship between Peripheral and Core Countries in the Euro Area using CDS returns
- propose a search algorithm aimed at the best separation of the variables
- with respect to maximum likelihood, the proposed method allows for a sufficient parametric flexibility
- Country spatial clustering changes by time, depending upon the intensity of the debt crises or how much strong common policies are shared among countries